Business Calculus Test 2 Review Answers

Dr. Graham-Squire, Summer Session 1, 2012

1. Use the limit definition of the derivative to calculate $f'(x)$ if $f(x) = \dfrac{1}{2x+3}$.

   Ans: You need to do $f(x+h) =$, etc. You answer should be $f'(x) = \dfrac{-2}{(2x+3)^2}$.

2. Find the derivative of each function:

   (a) $f(x) = (3x^4 - 7)(x^2 + 9)$. Use the product rule to get $18x^5 + 108x - 14x$

   (b) $f(x) = (x^3 - 7x + 9)^7$. Chain rule: Ans: $7(x^3 - 7x + 9)^6(3x^2 - 7)$

   (c) $f(x) = \left(\dfrac{x^3 - 9}{x + 4}\right)^3$. Ans: $\dfrac{3(x^3 - 9)^2(2x^3 + 12x^2 + 9)}{(x+4)^4}$

   (d) $f(x) = (x + 7)^4(3x^2 - 4)^2$. Ans: $4(x + 7)^3(3x^2 - 4)(6x^2 + 21x - 4)$

3. The quantity x of TV sets demanded each week is related to the wholesale price by the equation $p = -0.006x + 180$. The weekly total cost for producing x sets is given by $C(x) = 0.00002x^3 - 0.02x^2 + 120x + 60,000$.

   (a) Find the revenue function $R(x)$ and the profit function $P(x)$.

   Ans: $R(x) = -0.006x^2 + 180x$, $P(x) = -0.00002x^3 + 0.014x^2 + 60x - 60,000$.

   (b) Compute the marginal revenue, cost, and profit functions.

   Ans: $R'(x) = -0.012x+180$, $C'(x) = 0.00006x^2-0.04x+120$, $P'(x) = -0.00006x^2+0.028x+60$

   (c) Compute $R'(2000)$, $C'(2000)$, and $P'(2000)$ and interpret your results. What does that information tell the company about how many TV sets they should produce?

   Ans: $R'(2000) = 156$, $C'(2000) = 280$, and $P'(2000) = -124$. At production of 2000 TV sets, the costs still exceed the revenues and the next TV made will not give any profit. Producing only 2000 TV sets is not good for profits.

4. The number of people receiving disability benefits from 1990 through 2000 is approximated by the function

   $$N(t) = 0.00037t^3 - 0.0242t^2 + 0.52t + 5.3 \qquad (0 \leq t \leq 10)$$

   where $N(t)$ is measured in units of a million and t is measured in years with $t = 0$ being 1990. Compute $N(8)$, $N'(8)$, and $N''(8)$ and interpret your results. What does that information tell you about what was happening with disability benefits at that time, and what might it imply for the future?

   Ans: 8.1 million, 200,000, and 130,000. This means that in 1998, 8.1 million people were receiving disability benefits, and that number was increasing by 200,000 a year. Since the second derivative is positive, the increase is likely to continue for the next few years as well.

5. Elmo and Cookie Monster are both leaving Sesame Street in their cars. Elmo leaves at noon and drives straight north at 35 mph. Cookie leaves an hour later (at 1 pm) and drives straight east at 40 mph. How fast are the two monsters moving away from each other at 3 pm?

   Ans: About 50.5 miles per hour.

6. Let f be the function defined by $y = f(x) = \dfrac{2x^2+1}{x+1}$. Find the differential of f and use it to find the approximate change in y if x changes from 1 to 1.1.

   Ans: $dy = \dfrac{2x^2+4x-1}{(x+1)^2} \cdot dx$, $dy = 0.125$.

7. For each function, find
   (a) the intervals where the function is increasing or decreasing,
   (b) any relative maximum or minimum points (if any),
   (c) the intervals where f is concave up or down, and
   (d) inflection points (if any). For fun, you can also
   (e) sketch a graph of the function from the information you found, then compare to what you get when you put it into a graphing calculator.

   (i) $f(x) = x^4 - 2x^2$
   (ii) $f(x) = x\sqrt{x-1}$

   Ans: (i) is increasing on (-1,0) and (1,∞), decreasing on $(-\infty,-1)$ and $(0,1)$. minimums at $(\pm 1,-1)$ and a max at (0,0). It is concave up on $(-\infty,-1/\sqrt{3})$ and $(1/\sqrt{3},\infty)$ and concave down on $(-1/\sqrt{3},1/\sqrt{3})$, with inflection points at $(\pm 1/\sqrt{3},-1/9)$.

   (ii) is always increasing, has no max or min. Concave down on $(1,8/3)$ and concave up on $(8/3,\infty)$, with inflection point at $(8/3, 8\sqrt{5}/3\sqrt{3})$.